IMPACT: International Journal of Research in Applied, Natural and Social Sciences (IMPACT: IJRANSS)
ISSN (P): 2347-4580; ISSN (E): 2321-8851
Vol. 5, Issue 5, May 2017, 69-80

ROBUST ESTIMATORS FOR MARSHALL-OLKIN EXTENDED BURR III DISTRIBUTION

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ABSTRACT

The parameter estimation of the Marshall– Olkin extended Burr III (MOEBIII) distribution, which is a generalization of the Burr III distribution is considered. The maximum likelihood (ML) estimation of the parameters of the MOEBIII distribution is introduced by Al-Saiari *et al.* (2016). However, ML is often used to estimate the parameters of the Burr III distribution; this method is very sensitive to the presence of outliers in the data.

This paper presents M-estimation as a robust method, based on the quantile function to estimate the parameters of the MOEBIII distribution for complete data with outliers. A simulation study and a real data are used to illustrate the performance of M-estimator and ML estimator. The numerical results show that the M-estimator, generally is appropriate than the ML, in terms of the bias and mean square error when there are outliers in the data.

KEYWORDS: Marshall– Olkin Extended Burr III, Maximum Likelihood, M-estimator, Robust estimator, Outliers

1. INTRODUCTION

In (1942) Burr introduced a family of distributions, in which, the Burr type III distribution is one of this family. The Burr type III distribution covers a wider region in the skewness and kurtosis plane, including several distributions such as the gamma family, Weibull family, lognormal family, and the bell-shaped beta distributions, see Tadikamalla (1980). It has been widely used as a model in many areas such as environmental, economics and reliability analysis.

Marshall and Olkin in (1997) proposed a new family of survival functions based on adding a parameter to obtain a family of distributions, this distribution is called Marshall –Olkin extended distribution.

Using the survival function of any distribution $\bar{F}(x)$, the survival function of the new distribution $\bar{G}(x;\alpha)$ can be defined as follows:

$$\bar{G}(x;\alpha) = \frac{\alpha\bar{F}(x)}{1-\bar{\alpha}\bar{F}(x)}, -\infty < x < \infty, 0 < \alpha < \infty \quad (1)$$

where $\alpha = 1 - \bar{\alpha}$ is an additional positive parameter and they have called the tilt parameter [See Marshall and Olkin (2007)]. The cumulative distribution (cdf) and the probability density function (pdf) for the new distribution are given respectively by:

$$G(x;\alpha) = \frac{F(x)}{1-\bar{\alpha}\bar{F}(x)} = 1 - \frac{\alpha\bar{F}(x)}{1-\bar{\alpha}\bar{F}(x)}, \quad (2)$$

$$g(x,\alpha) = \frac{\alpha f(x)}{[1-\bar{\alpha}\bar{F}(x)]^2} \quad (3)$$

There have been various studies reported in the literature dealing with the parameter estimation methods for Burr III distribution. Lindsay *et al.* (1996) compared between the Burr type III distribution with four parameters and the Weibull distribution using the diameter data. Shao (2000) investigated the constrained ML method to estimate the parameters of the Burr type III distribution for toxicity data. Shao *et al.* (2008) used of the extended Burr type III distribution under the re-parameterization method in low-flow frequency analysis, where the lower tail of the distribution is of interest and used three methods of estimation, method of moments, probability-weighted moments, and maximum likelihood method to estimate the unknown parameters of the distribution.

An alternative robust estimation method based on M-estimators and AM-estimators estimation method for the parameters of Burr III distribution have been proposed by Wang and Wen Lee (2010, 2011 and 2014) respectively as follows:

In (2010) they used the M- estimator to estimate the parameters of the extended three- parameter Burr type III distribution for complete data with outliers and compared it with ML and least squares methods. In (2011) they introduced M- estimator and ML and AM- estimator, when the data is asymmetric to estimate the parameters of the Burr type III distribution for complete data with outliers and compared the results. Also, in (2014) they discussed the M-estimator for estimating the Burr type III parameters with outliers and compared it with ML and least squares methods.

However, there is not much work concerning MOEBIII distribution. The parameters of MOEBIII distribution was estimated by using ML estimation method by Al-Saiari *et al.* (2016). However, it is well established that, in the presence of outliers in the data, the traditional methods do not provide reliable estimators. Therefore, robust estimation methods can be used for estimating the parameters of the MOEBIII distribution, if the data contains outliers.

This paper is organized as follows: In Section 2, the Marshall– Olkin extended Burr III (MOEBIII) distribution is described. In Section 3, the maximum likelihood and the robust M-estimator are introduced. A simulation study and a real data to illustrate the performance of ML estimator and M-estimator are illustrated in Sections 4 and 5, respectively. Finally, concluding remarks are given in Section 6.

2. MARSHALL-OLKIN EXTENDED BURR III DISTRIBUTION

The cumulative distribution (cdf) and the probability density function (pdf) for a two shape parameters $c > 0$, $k > 0$ Burr type III distribution are given by:

$$F(x; c, k) = (1 + x^{-c})^{-k}\ ;\ x \geq 0 \quad \textbf{(4)}$$

and

$$f(x; c, k) = k\, c\, x^{-(c+1)}(1 + x^{-c})^{-(k+1)};\ x \geq 0 \quad \textbf{(5)}$$

where c, k are shape parameters.

Substituting (4) and (5) in (2) and (3) the cdf and the pdf of MOEBIII are given respectively as:

$$G(x; \alpha, c, k) = \frac{(1+x^{-c})^{-k}}{\left[1-(1-\alpha)\left[1-(1+x^{-c})^{-k}\right]\right]};\ x \geq 0 \quad \textbf{(6)}$$

and

$$g(x;\alpha,c,k) = \frac{\alpha\, k\, c\, x^{-(c+1)}(1+x^{-c})^{-(k+1)}}{\left[1-(1-\alpha)[1-(1+x^{-c})^{-k}]\right]^2}; x \geq 0 \tag{7}$$

where, α is called tilt parameter, when $\alpha = 1$, the original distribution i.e., Burr type III distribution with two parameters is obtained.

MOEBIII distribution is more flexible than the BurrIII distribution, because of the presence of the additional shape parameter. Figure 1 shows the plots of pdf for MOEBIII distribution for some values of the parameters. [See Al-Saiari.*et.al.* (2016)]

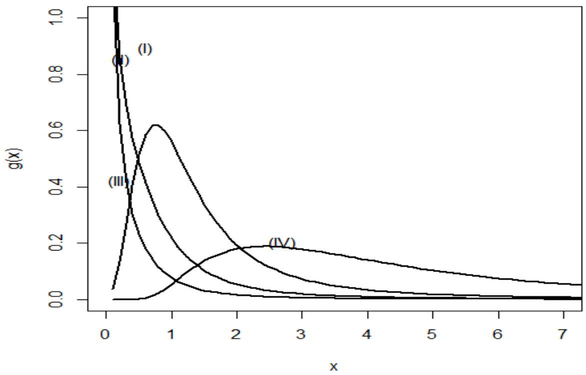

Figure 1: Plot of the PDF of the MOEBIII distribution. (I) α = 0.8, k=0.1, c=2, (II) α = 3, k=0.1, c=2, (III) α = 0.8, k = 5, c = 2, (IV) α = 3, k = 5, c = 2

The qth quantile of the MOEBIII is given by

$$x_q = G^{-1}(q) = \left[\left[\left(\frac{1-q}{\alpha q}\right) + 1\right]^{1/k} - 1\right]^{-1/c}; \; 0 \leq q \leq 1 \tag{8}$$

3. PARAMETERS ESTIMATION

In this section, ML estimation method to estimate the parameters of the MOEBIII distribution is first reviewed. The robust estimators based on the M-estimation method given by Huber (1964) are introduced.

3.1 The Maximum Likelihood Method

Suppose that $\boldsymbol{X} = (x_1, x_2, \dots, x_n)$ is a random sample of size n from MOEBIII (α, c, k), then the log-likelihood function $l(\alpha, c, k)$ is given by:

$$l(\alpha,c,k) = n(log\alpha + logc + logk) - (c+1)\sum_{i=1}^{n} log(x_i) - (k+1)\sum_{i=1}^{n} log(1+x_i^{-c}) - 2\sum_{i=1}^{n} 1 - (1 - \alpha)\,[1-(1+x_i{}^{-c})^{-k}] = 0 \tag{9}$$

To obtain ML estimates of the parameters, equation (9) is maximized by taking the partial derivatives with respect to α, c and k setting it to 0 as follows:

$$\frac{\partial l(\alpha,c,k)}{\partial \alpha} = \frac{n}{\hat{\alpha}} - 2\sum_{i=1}^{n} \frac{1-(1+x_i^{-\hat{c}})^{-\hat{k}}}{1-(1-\hat{\alpha})\left[1-(1+x_i^{-\hat{c}})^{-\hat{k}}\right]} = 0 \qquad \textbf{(10)}$$

$$\frac{\partial l(\alpha,c,k)}{\partial c} = \frac{n}{\hat{c}} - \sum_{i=1}^{n} log(x_i) + (\hat{k}+1)\sum_{i=1}^{n} \frac{x_i^{-\hat{c}} log(x_i)}{(1+x_i^{-\hat{c}})} -$$

$$2(1-\hat{\alpha})\hat{k}\sum_{i=1}^{n} \frac{x_i^{-\hat{c}} log(x_i)(1+x_i^{-\hat{c}})^{-(\hat{k}+1)}}{1-(1-\hat{\alpha})\left[1-(1+x_i^{-\hat{c}})^{-\hat{k}}\right]} = 0 \qquad \textbf{(11)}$$

$$\frac{\partial(\alpha,c,k)}{\partial k} = \frac{n}{\hat{k}} - \sum_{i=1}^{n} log(1+x_i^{-\hat{c}}) + 2(1-\hat{\alpha})\sum_{i=1}^{n} \frac{(1+x_i^{-\hat{c}})^{-\hat{k}} log(1+x_i^{-\hat{c}})}{1-(1-\hat{\alpha})\left[1-(1+x_i^{-\hat{c}})^{-\hat{k}}\right]} = 0 \qquad \textbf{(12)}$$

Since equations (10) - (12) are non-linear equations, it cannot be solved analytically; numerical methods should be used to obtain the ML estimates.

3.2 M- Estimation Method

The robust M-estimator method is used to estimate the parameters of the MOEBIII distribution from a sample of n observations$(x_1, x_{2,} \dots, x_n) = x^T$. As follows:

$$x_i = f_i(\theta) + u_i; i = 1, \dots, n \qquad \textbf{(13)}$$

where $\theta^T = (\alpha, c, k)$, $f_i(\theta)$ is the quantile function of the MOEBIII distribution and u_i is the error term with mean equals zero and variance equals σ^2. Gilchrist (2000) introduced the robust property based on the quantile function. For the robust regression method, it is necessary to scale the invariant error, as follows:

$$e_i = \frac{u_i}{s} \qquad \textbf{(14)}$$

where $s = \frac{MAD(u)}{0.6745}$ and $MAD(u) = MAD(u_1, u_2, \dots, u_n) = Median\{|u - Median(u)|\}$

[See Hampel *et.al.* (1986)]

The M-estimator method for estimating the MOEBIII parameters is defined by minimizing the objective function of all invariant errors, $\rho(e_i)$ as follows:

$$Minimize \sum_{i=1}^{n} \rho(e_i) \qquad \textbf{(15)}$$

To estimate the three unknown parameters of the MOEBIII distribution, a simple comparison among two different ρ objective functions is used. The selected objective functions areTukey's Bisquare and Huber's weight [See Huber (1981)].

Tukey's Bisquare objective function is derived as:

$$\rho(e_i) = \begin{cases} \frac{a^2}{6}\left(1-(1-\left(\frac{e_i}{a}\right)^2\right)^3 & if\ |e_i| \le a \\ \frac{1}{6}a^2 & if\ |e_i| > a \end{cases} \qquad \textbf{(16)}$$

with the derivative

$$\psi(e_i) = \begin{cases} e_i\left((1-\left(\frac{e_i}{a}\right)^2\right)^2 & if\ |e_i| \le a \\ 0 & if\ |e_i| > a \end{cases}$$

where a= 4.685 and e_i is the scale invariant error in (14).

Huber's weight objective function is derived as:

$$\rho(e_i) = \begin{cases} \frac{1}{2}(e_i)^2 & if\ |e_i| < a \\ a|e_i| - \frac{1}{2}a^2 & if\ |e_i| \ge a \end{cases} \quad \textbf{(17)}$$

with the derivative

$$\psi(e_i) = \begin{cases} e_i & if\ |e_i| < a \\ a\ sign\ e_i & if\ |e_i| \ge a \end{cases}$$

where a= 1.345 and e_i is the scale invariant error in (14).

The estimators of the parameters can be obtained for the two selected objective functions are derived by differentiating (15) with respect to $\alpha, c\ and\ k$respectively, and equating to zero. Then, the simultaneous equations can be obtained, as follows:

$$\sum_{i=1}^{n}\psi(e_i)\frac{\partial f_i}{\partial \alpha} = \sum_{i=1}^{n}\psi(e_i) \times \frac{-1}{\hat{c}\hat{k}}\left[\left[\left(\frac{1-q}{\hat{\alpha}q}\right)+1\right]^{\frac{1}{\hat{k}}} - 1\right]^{-\left(\frac{1}{\hat{c}}+1\right)}$$

$$\times\left[\left(\frac{1-q}{\hat{\alpha}q}\right)+1\right]^{\frac{1}{\hat{k}}-1}\left[\frac{-(1-q)}{\hat{\alpha}^2 q}\right] = 0 \quad (18)$$

$$\sum_{i=1}^{n}\psi(e_i)\frac{\partial f_i}{\partial c} = \sum_{i=1}^{n}\psi(e_i) \times \frac{1}{\hat{c}^2}\left[\left[\left(\frac{1-q}{\hat{\alpha}q}\right)+1\right]^{\frac{1}{\hat{k}}} - 1\right]^{-\frac{1}{\hat{c}}}$$

$$\times\ log\left[\left[\left(\frac{1-q}{\hat{\alpha}q}\right)+1\right]^{\frac{1}{\hat{k}}} - 1\right] = 0 \quad (19)$$

$$\sum_{i=1}^{n}\psi(e_i)\frac{\partial f_i}{\partial k} = \sum_{i=1}^{n}\psi(e_i) \times \frac{1}{\hat{c}\hat{k}^2}\left[\left[\left(\frac{1-q}{\hat{\alpha}q}\right)+1\right]^{\frac{1}{\hat{k}}} - 1\right]^{-\left(\frac{1}{\hat{c}}+1\right)}$$

$$\times\left[\left(\frac{1-q}{\hat{\alpha}q}\right)+1\right]^{\frac{1}{\hat{k}}} log\left[\left(\frac{1-q}{\hat{\alpha}q}\right)+1\right] = 0 \quad (20)$$

Since equations (18)-(20) are non-linear equations, numerical iterative methods can be used to obtain the M-estimators for the parameters $\alpha, c\ and\ k$.

4. SIMULATION STUDY

In this section, a simulation study to compare the performance of the ML and the robust M-estimators is carried out in the presence of outliers. The data were generated from the MOEBIII distribution for several different values of

$\theta^T = \alpha, c$ and k.

The steps of simulation study are given as:

- Step 1. Generate n random samples $w_1, w_2, \ldots, w_n$ from uniform distribution ~U (0, 1).
- Step 2 Generate a random sample x_i from the MOEBIII distribution for specified values of $\alpha, c\ and\ k$ distribution using equation (8) for different values of $i, i = 1,2, \ldots, n$.
- Step 3. Select different sample sizes, n = 20, 40,100 to represent the small, moderate and the large sample sizes, respectively.
- Step 4. The outliers are generated for each random sample from the uniform distribution as U ($\bar{X}$+ 4S, $\bar{X}$ + 7S), where $\bar{X}$ the samples mean of is $X = (x_1, x_2, \ldots, x_n)$ and S is the sample standard deviation of X. The outliers are generated by shifting the largest observations to the right in the X direction. For the small sample size (n = 20) one outlier is taken, for the moderate sample size (n = 40) two outliers are taken and for the large sample size (n = 100) five outliers are taken. [See Wei and Fung (1999)].
- Step 5. Different initial parameter values for α, c and k are taken.
- Step 6. Obtain the ML estimates using equations (10) - (12).
- Step 7. Obtain the M- estimates using equations (18)-(20).
- Step 8. calculate the bias and mean square errors (MSE) of the estimates derived in the steps 6 and 7 using the following formulae:

  $Bias(\hat{\theta}) = E(\hat{\theta}) - \theta$

  $MSE = \frac{1}{N}\sum_{i=1}^{N}(\hat{\theta} - \theta)^2$

  where,$E(\hat{\theta}) = \frac{1}{N}\sum_{i=1}^{N}\hat{\theta}$
- Step 9. Repeat steps (1-7), N=1000 time.

The results in Tables 1-3 clearly indicates that, the robust estimator based on Tukey's Bisquare function has the smallest bias and the smallest MSE in most of the cases are with outliers. In addition, the robust estimator based on Huber's function outperforms ML estimator in terms of bias and MSE when the data set contains outliers. For example, it can be seen from Tables 1-3 that, the largest difference of the bias for parameter c arises in Table 1 for the cases (0.2, 2.0, 0.2) and (0.3, 2.0, 0.1). Similarly, one can observe superiority of the robust estimators in terms of bias and MSE in Tables 1-3. Moreover, in Table 3 it can be noted that when the number of outliers increase, the ML estimators increase when compared to the robust estimators. It can be seen that, bias and the MSE values are very large for the ML estimators. In general, it is advised to use the MOEBIII distribution instead of using classical methods when there are potential outliers in the data.

Table 1: The Bias and MSE (in Parenthesis) for n=20 with One Outlier

Parameter α			
Parameters	**ML**	**Tukey's Bisquare**	**Huber**
(0.1,2.0,0.1)	0.2695 (0.1485)	0.2735 (0.1057)	0.2653 (0.1086)
(0.2,2.0,0.2)	0.2207 (0.1146)	-0.0855 (0.0267)	0.1357 (0.0732)
(0.3,2.0,0.3)	0.1097 (0.1038)	- 0.1758 (0.0419)	-0.0239 (0.0528)
(0.3,2.0,0.1)	0.2470 (0.1225)	-0.0606 (0.0502)	0.1884 (0.0926)
(0.8,2.0,0.1)	-0.1333 (0.0742)	-0.0279 (0.0043)	-0.1647 (0.0682)
Parameter c			
	ML	**Tukey's Bisquare**	**Huber**
(0.1,2.0,0.1)	-0.3315 (6.2178)	-0.47301 (0.7879)	-0.0661 (0.2523)
(0.2,2.0,0.2)	1.3283 (41.1222)	0.2037 (9.3444)	-0.1857 (0.6362)
(0.3,2.0,0.3)	-0.2962 (5.4922)	-0.3118 (1.5008)	-0.62901 (1.0165)
(0.3,2.0,0.1)	1.1688 (32.7146)	0.9089 (11.9765)	0.3605 (5.2050)
(0.8,2.0,0.1)	0.0501 (11.1791)	-0.3109 (3.2648)	-0.7966 (0.8742)
Parameter k			
	ML	**Tukey's Bisquare**	**Huber**
(0.1,2.0,0.1)	0.6996(2.4519)	-0.0316 (0.0019)	0.1019 (0.2715)
(0.2,2.0,0.2)	0.0545 (0.0638)	-0.0772 (0.0122)	0.0601 (0.0151)
(0.3,2.0,0.3)	0.4316 (1.0632)	0.0856 (0.2068)	0.3009 (0.6877)
(0.3,2.0,0.1)	0.0418 (0.0296)	-0.0083 (0.0086)	0.0167 (0.0078)
(0.8,2.0,0.1)	0.0427 (0.0159)	-0.0279 (0.0043)	0.03561 (0.0055)

Table 2: The Bias and MSE (in Parenthesis) for n= 40 with 2 Outliers

Parameter α			
parameters	**ML**	**Tukey's Bisquare**	**Huber**
(0.1,2.0,0.1)	0.1162 (0.0319)	0.0876 (0.0293)	0.1421 (0.0218)
(0.2,2,0.2)	0.3606 (0.1803)	0.0022 (0.0556)	0.1483 (0.0615)
(0.3,2,0.3)	0.1445 (0.0904)	-0.0557 (0.0474)	0.0327 (0.0527)
(0.3,2.0,0.1)	0.2459 (0.1024)	-0.0031 (0.0651)	0.1805 (0.0729)
(0.8,2.0,0.1)	-0.4973 (0.3170)	-0.1038 (0.0552)	-0.3035 (0.1660)
Parameter c			
	ML	**Tukey's Bisquare**	**Huber**
(0.1,2.0,0.1)	0.2669 (11.7336)	-1.2805 (1.7296)	-0.9759 (1.1687)
(0.2,2,0.2)	-1.2267 (1.7687)	-0.6979 (0.7243)	-0.5429 (0.4339)
(0.3,2,0.3)	-0.3030 (1.2793)	-0.9431 (1.1152)	-0.8058 (1.2048)
(0.3,2.0,0.1)	0.3482 (11.8764)	-0.4349 (3.1586)	-0.3435 (1.6942)

Table 2: Contd.,			
(0.8,2.0,0.1)	-0.4489 (1.9466)	-0.8765 (1.6520)	-0.6734 (1.1204)
Parameter k			
	ML	**Tukey's Bisquare**	**Huber**
(0.1,2.0,0.1)	0.1368 (0.0356)	0.0386 (0.0083)	0.1030 (0.0202)
(0.2,2,0.2)	0.10367 (0.0286)	-0.0399 (0.0089)	0.0076 (0.0027)
(0.3,2,0.3)	0.1186 (0.1028)	-0.1806 (0.0472)	0.0555 (0.0554)
(0.3,2.0,0.1)	0.0768 (0.0269)	0.0162 (0.0208)	0.0555 (0.0095)
(0.8,2.0,0.1)	0.1057 (0.0435)	-0.0074 (0.0054)	0.0998 (0.0217)

Table 3: The Bias and MSE (in Parenthesis) for n=100 with 5 Outliers

Parameter α			
Parameters	**ML**	**Tukey's Bisquare**	**Huber**
(0.1,2.0,0.1)	0.2608 (0.1478)	-0.0199 (0.0004)	0.1808 (0.0696)
(0.2,2.0,0.2)	0.1126 (0.0530)	-0.0453 (0.0375)	0.0683 (0.0239)
(0.3,2.0,0.3)	0.1207 (0.0581)	-0.0991 (0.0319)	0.04755 (0.0410)
(0.3,2.0,0.1)	0.2354 (0.0928)	0.0067 (0.0442)	0.1290 (0.0397)
(0.8,2.0,0.1)	-0.4048 (0.2387)	-0.0191 (0.0205)	-0.0249 (0.0213)
Parameter c			
	ML	**Tukey's Bisquare**	**Huber**
(0.1,2.0,0.1)	0.4732 (4.2175)	-0.0575 (1.9262)	0.2640 (1.50503)
(0.2,2.0,0.2)	-1.2290 (1.8074)	-0.7163 (0.5969)	-0.7037 (0.8123)
(0.3,2.0,0.3)	-0.7874 (1.0200)	-0.6787 (0.7803)	-0.6569 (0.7127)
(0.3,2.0,0.1)	0.0947 (5.3904)	-0.6648 (1.1848)	--0.2354 (2.1846)
(0.8,2.0,0.1)	-1.1227 (1.5925)	-0.7149 (0.6458)	-0.7023 (0.8461)
Parameter k			
	ML	**Tukey's Bisquare**	**Huber**
(0.1,2.0,0.1)	0.7522 (3.1906)	0.0112 (0.0072)	0.0776 (0.0258)
(0.2,2.0,0.2)	0.1934 (0.0923)	0.0844 (0.0332)	0.2331 (0.1117)
(0.3,2.0,0.3)	0.2987 (0.7993)	0.0035 (0.2025)	0.0965 (0.0278)
(0.3,2.0,0.1)	0.0621 (0.0219)	-0.0179 (0.0033)	0.0513 (0.0109)
(0.8,2.0,0.1)	0.0833 (0.0102)	-0.0068 (0.0046)	0.0606 (0.0089)

5. APPLICATION TO REAL DATA

In this section, the data set given by Shao (2000) is used. This data set is used by Wang and Wen Lee (2011 and 2014), to illustrate the Burr type III distribution and the ability of the M- estimation method and AM- estimation method. This data are the result of the study of influence of the proportion of toxicity of chromium in marine waters.

The data set, which is given in Table 4, contains 36 for chromium in marine waters. The box plot of the dataset given in Figure 2 displays a potential outlier in the data set.

Table 4: Number Observed Effect Concentration (NOEC) Values for Chromium in Marine Water

Marine Water
2000, 12.59, 199.53, 4, 56, 263.03, 776.25, 1258.93, 3311.31, 39.81, 2630.27, 187.2, 89.13, 4.79, 8800, 10000, 1600, 602.56, 2000, 199.5, 2.4, 728, 1122.02, 140, 177.83, 540, 1456, 2511.89, 1200, 478.63, 1174.89, 9.55, 3090.3, 602.56, 1995.26, 210.

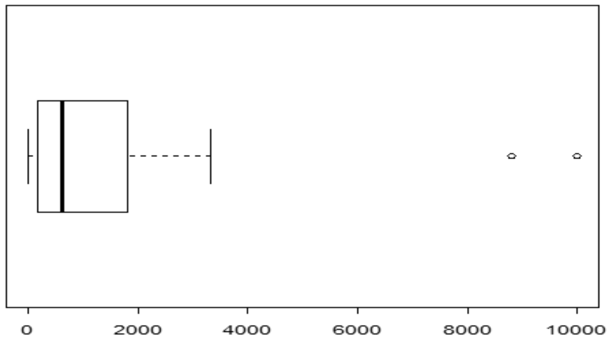

Figure 2: Box plot of the Chromium in Marine Water Data

The data set is used to fit a MOEBIII distribution and to estimate the unknown parameters α, c and k, using the ML and the M-estimator based on two different objective functions:Tukey's Bisquare and Huber's function. Table 5 gives summary of fitting the MOEBIII distribution obtained from the ML, and the robust estimation methods for this data set. The 95% confidence intervals of α, c and k for ML and M-estimator with the two objective functions, Tukey's Bisquare and Huber's are shown in Table 5. In Table 5, one can see that, the confidence intervals of Tukey's Bisquare are estimates are better than the other fitted obtained from the Huber and ML estimates. Also, it is found that Huber estimates is better than the ML estimates.

Table 5: Parameter estimates, Standard Deviation (in Parentheses) and Confidence Intervals [in Parentheses] for the data Set

Parameters	Method		
	ML	Tukey's Bisquare	Huber
α	5.1 (0.852) [3.43,6.77]	5.9 (0.110) [5.784,6.215]	5.6 (0.273) [5.065,6.135]
c	0.65 (0.0739) [0.40,0.795]	0.71(0.035) [0.641,0.779]	0.59 (0.0583) [0.476, 0.704]
k	7.8 (1.705) [5.66,11.14]	8.9(0.098) [8.708,9.092]	8.3 (0.197) [7.914,8.686]

6. CONCLUSIONS

The M-estimator method based on the quantile function by the Tukey's Bisquare and Huber objective function to estimate the MOEBIII parameters for complete data with outliers are presented. The simulation results showed that the M-estimator method out performs the ML method regarding bias and mean square error. The real data showed that the M-estimator method is suitable for estimating the MOEBIII parameters for complete data with outliers. Also, the confidence intervals of the MOEBIII parameters for M-estimator and ML are obtained using this data set. The confidence intervals of Tukey's Bisquare estimates are comparably better than the other fitted, obtained from the Huber and ML estimates. The Huber estimates is also better than the ML estimates. In general, it is advised to use the MOEBIII distribution instead of using classical methods, when there are potential outliers in the data.

7. ACKNOWLEDGEMENTS

The author thanks the editors for their comments which improved the original version of the paper.

8. REFERENCES

1. Al-Saiari, A. Y., Baharith, L. A. and Mousa, S. A. (2016). Marshall-Olkin Extended Burr III Distribution. International Mathematical Forum. Vol. 11, (13), 631 – 642
2. Burr I. W. (1942). Cumulative Frequency Functions. The Annals of Mathematical Statistics. Vol. 13, (2), 215–232.
3. Gilchrist, W.G. (2000). Statistical Modeling with Quantile Function, Chapman & Hall, New York.
4. Hampel F.R., Ronchetti E.M., Rousseeuw P.J. and Stahel W.A. (1986). Robust Statistics: The Approach Based on Influence Functions, Wiley, New York.
5. Huber, P. J. (1964). Robust Estimation of a Location Parameter. The Annals of Mathematical Statistics, Vol. 35, 73-101.
6. Huber, P.J. (1981). Robust Statistics, Wiley, New York.
7. Lindsay, S.R., Wood, G.R. and Woollons, R.C. (1996). Modeling the Diameter Distribution of Forest Stands Using the Burr Distribution. Journal of applied statistics, Vol. 23, (6), 609-619.
8. Marshall, A. W. and Olkin, I., (1997). A New Method for Adding a Parameter to a Family of Distributions with Application to the Exponential and Weibull Families. Biometrika, Vol. 84, (3), 641-652.
9. Marshall A.W. and Olkin I. (2007). Life Distributions. Structure of Nonparametric, Semiparametric and Parametric Families. Springer, New York.
10. Shao, Q. (2000). Estimation for Hazardous Concentrations Based on NOEC Toxicity Data: an Alternative Approach. Environmetrics, Vol. 11, 583–595.
11. Shao Q., Chen Y. D. and Zhang L. (2008). An Extension of Three-Parameter Burr III Distribution for Low-Flow Frequency Analysis. Computational Statistics and Data Analysis, Vol. 52, (3), 1304–1314.
12. Tadikamalla, P.R. (1980). A look at the Burr and Related Distributions. International Statistical Review, Vol. 48, (3), 337–344.
13. Wang F. K. and Lee C.-W. (2010). A M-Estimator for Estimating the Extended Burr Type III Parameters with Outliers. Communications in Statistics -Theory and Methods, Vol. 40, (2), 304–322.

14. Wang F. K. and Lee C.-W. (2011). M-estimator with Asymmetric Influence Function for Estimating the Burr type III Parameters with Outliers. Computers and Mathematics with Applications, Vol. 62, (4), 1896–1907.

15. Wang F. K. and Lee C. W. (2014). M-estimator for Estimating the Burr Type III Parameters with Outliers. Mathematics and Computers in Simulation, Vol. 105, (4), 144–159.

16. Wei W.H. and Fung W.K. (1999). The Mean-Shift Outlier Model in General Weighted Regression and its Application, Computational Statistics & Data Analysis, Vol. 30, 429–441.